Global Journal of Modeling and Intelligent Computing (GJMIC)

Vol. 1, No. 1, January, 2021 ISSN: 2767-1917

RELIABILITY AND PERFORMANCE ANALYSIS OF MARKOVIAN FAULT TOLERANT SYSTEM WITH VACATION

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ABSTRACT

In many real-world applications, reliability analysis plays an extremely significant role. This investigation analyses the reliability features of the fault-tolerant system with online and standby units, including the concept of vacation and reboot. In this study, the threshold recovery policy is taken into account to utilize the repair facility effectively. If a failed device is not spotted, the system will enter into an unstable state, where the system will automatically reconfigure by reboot process to eliminate the faults. The system failures may not successfully be fixed due to imperfect switching. The server will go on vacation when there are no failed machines waiting for a repair job, and the server returns back from vacation whenever there are pre-defined units waiting to be repaired in the system. The transient equations are established forthe system states and solved by determining the eigenvalues of the transition matrix. The system metrics along with reliability and mean time to failure (MTTF) are acquired. A variety of cases are examined to study the impact on system reliability and MTTF with respect to different parameters of the system. Numerical results for performance evaluation of the system and cost optimization are provided by taking an illustration.

Keywords: Machining system, Vacation, N-policy, Reboot, Sensitivity analysis, Reliability, MTTF.

1. INTRODUCTION

Failures of the machining parts have a huge effect on the functioning and performance of fault-tolerant systems (FTS) involved in computers, communications, production systems and a number of certain other systems. Occurrence of faults in machining environment results in a loss of desired efficiency and output, as well as an increase in downtime and expenditure. Some units may have malfunctions in FTS, but due to maintenance, optimum control, and standby, the system is operational and manages to do its required tasks. As modern technology progresses, FTSs plays a critical role in making the device more functional and fault-tolerable. The idea of fault tolerance with reboot processes has caught the considerable interest of both researchers and academicians involved in the development and designing of the machining system in order to achieve significant reliability.

In order to upgrade the service and maintenance strategies for the FTS, the quantitative evaluation of different transient metrics can be introduced by implementing the concepts of threshold recovery, working breakdown, vacation and standby. Recently, Kumar and Jain (P. Kumar & Jain, 2020) investigated an FTS with working breakdown, including the concept of reboot and recovery. This study is further extended for the transient behavior of FTS, which incorporates the features of vacation, N-policy, working breakdown, threshold recovery along with reboot and recovery. Since, no study has occurred in the literature including these concepts. The proposed Markov model is applicable in many real-time systems, including networks, production lines, communication systems, service providers, power plants, etc. The transient analysis of standby-supported machining system in generic set up is done by framing the governing equations for the transient state. In order to evaluate the reliability and resolve the maintenance issues of the standby-supported FTS, the main objective of the present investigation is to accomplish the objectives:

(i) Practical features of unreliable server, standby support, reboot, imperfect switching and vacation in the finite population model.

(ii) Laplace transformation of the transition matrix is taken to create the analytical expressions for both the reliability function $R_Y(\tau)$ and MTTF. The validations of the computational performance indices as well as the sensitivity with regard to the parameters of the device are done.

The contents of the current study are arranged as follows in various sections. The related literature to the developed model is presented in Section 2. The model description and assumptions are reported in Section 3 to formulate the transient Markov model for FTS. The model governing equations are framed in Section 4. The matrix method approach is given in Section 5 to evaluate the transient distribution of system states. Section 6 establishes the performance metrics and sensitivity analysis. Section 7 presents the numerical results to explore different system parameters with variations in parameters. In Section 8, the results of the model under review are summarized and concluded.

2. LITERATURE REVIEW

The computer-controlled manufacturing technology implemented enormous modifications in the machining scheme to control system failure. Many electrical and electronic devices are designed with an integrated fault handling device that automatically identifies system failures and restores the system by using the backup unit to substitute the failed unit. Few devices may be temporarily rebooted, if the fault handling system is unsuccessful in identifying and restore the fault. The fault can no longer be retrieved by the fault handling tool in many sensitive conditions; such conditions are called imperfect coverage. To investigate the performance of a machining system with reboot and imperfect coverage, Ke et al.(2008)used a Bayesian approach. The outstanding work on machining systems that incorporates the reboot idea and imperfect coverage has been done by many researchers (cf. Ke and Liu(2014), Yang et al.(2015)). Jain (2013)examined the characteristics of the machining system to demonstrate the mathematical tractability of the generated analytical results. Jain and Meena(2017)analyzed the transient behavior of a fault-tolerant system with spare part support using the Runge-Kutta method. Ke and Liu (2014)considered the modeling of a repairable system in which reboot, repair, and failure times are considered to be exponential. They performed the sensitivity analysis of the system for various distributions.
It is often an essential requirement to maintain quality and a high level of reliability. The survey articles on the study of machining systems via queueing theory by Haque et al. (2007), Kolledath et al. (2018) provide an overview of the queueing models on machining systems with spare support. The efficiency of a machining system has been examined by Shekhar et al. (2017) by incorporating the concept of switching failure and geometric reneging. The relative sensitivity analysis was provided to explore the system descriptors for the technical justification and relevance of the established results. An unreliable queueing system with imperfect standby and unpredictable breakdowns has been addressed by Ke et al. (2018). Ke et al. (2016) conducted the performance modeling of the machining system with spares. They framed the system efficiency metrics and estimated the cost function to minimize the cost of the system. Wang et al. (2013)conducted a study of reliability and sensitivity with imperfect coverage for a repairable device. They attained the reliability indices of the system related to system's parameters. The system's reliability has been examined by various researchers (Ke et al. (2007), Kuo et al. (2014)). Jain and Kumar (2018) analyzed the reliability of two models and also completed the results obtained using ANFIS.
Over the last decade, an enormous consideration has been paid to the applications of optimal control policies. N-policy states that when ' N ' or more clients are gathered in the system, the server is switched on and switched off when no more jobs are waiting in the system. Jain et al. (2014) conducted the transient analysis of an unreliable server machining system composed of operating units and standbys. To address a more realistic situation, they incorporated the concept of N-policy. Jain et al. (2004)suggested N-policy to study the reliability metrics of machine repair systems (MRS). The numerical analysis of a queueing system with threshold recovery for the transient state behavior has been carried out by Ezeagu et al. (2018) using R-K method.
In threshold recovery policy, the failed units are repaired after the accumulation of some repair jobs i.e., when the failed units reach a level 'q', the repair is started. In the modern technology age, the threshold policy can be used to prevent the loss of valuable resources, money, and time in the machining system. Several queue researchers have paid their attention to the issue of system repairs under different conditions. Jain and Bhagat(2012) examined the system's performance measures of MRP with the unreliable server by considering the concept of threshold recovery. Kumar et al. (2018)developed an F-policy for machining system, and established the Markov model using the mechanism of birth-death. To calculate the transient probability of the system, a matrix approach has been used, and the system's cost is optimized.

In the modeling of machining systems, the concept of server vacations has also been incorporated in some research works. Yang and Tsao(2019)investigated a device in which the repairman takes multiple vacations when there are no system components in a broken-down state. They used the matrix-analytical methods to evaluate the steady-state metrics and provided numerical results to determine the system parameter's effect on the reliability characteristics of the system. In work performed by Osaki and Nakagwa(1976), a thorough overview of reliability analysis of the MRS is available. Meena et al.(2019)obtained a vacationing server machining system with standby and determined the functional usefulness of the examined model. Wang et al. (2009)investigated a finite capacity machining system with working vacation, according to which the server is operating at a slower repair rate instead of stopping the repair job completely.

Faults in the service provider can often occur while the system is under operation. It is a commonly used assumption for the analysis of MRP that the service is not provided by the repairman when the server breakdown. The concept of working breakdown during a minor breakdown period was initiated by Kalidass and Kasturi(2012). He mentioned that the server operates during the breakdown state in low capacity instead of stopping the service completely. Rajadurai(2018) carried out the sensitivity analysis of a queueing system having the provision of re-attempts and working breakdowns. An unreliable queueing system with delays in the repair process was discussed by Choudhury and Tadj(2009). An unreliable Markovian queue with working breakdowns has been analyzed by Liou(2015)by employing the matrix-geometric approach. The concept of system's recovery to analyze a machining system with working breakdown has been used by Chen(2018).They examined the reliability analysis of the system and executed some numerical experiments. A Markovian queueing system was studied by Yang and Wu (2017), including the concept of working breakdowns, repair, and reneging. Yen et al. (2016)analysed an unreliable machining system.

3. MODEL DESCRIPTION

To evaluate the performance of an FTS consisting of M operating units and S standby units, a Markov model can be used. The practical features including N-policy and threshold recovery are considered while developing the Markov model. The server is unreliable, and there is a provision of standbys supports and maintenance via repair of the failed units.

3.1 Assumptions and Notations

The underlying assumptions are considered in the formulation of FTS consisting of M active and S spare i.e., passive units. The units that are operational are vulnerable to failure; the lifetime of the units is distributed exponentially with the rate $\lambda (0<\gamma<\lambda)$. As the active unit fails, it is automatically substituted by an accessible spare unit.

Table 1: Notations used in the model formulation of FTS

λ	The failure rate of operating units
r	Reboot rate
ψ	N-policy
ε	Vacation rate
μ	Repair rate of failed machines
c	Probability coverage of failed operating unit
$1/\alpha$	Mean lifetime of the server
$1/\beta$	Mean repair time of the server
MTTF	Meantime to failure
M	Total active units in the system
$R_Y(\tau)$	System reliability
L(S)	Identical online (Standby) machines

If the spare unit switches over to the operational state, we presume that its failure characteristics match with the active unit. The failure occurs in the same sequence in which the repair of the broken machines is carried out, i.e.,the repair is performed in accordance with the first come first served (FCFS)discipline. It is assumed that the failed machines are repaired according to an exponential distribution at a rate μ. Whenever no broken-down units are waiting in the system for getting repaired, the repairman is able to go on vacation; the duration of vacation is generated by the exponential distribution with rate ε. After a pre-determined period of time following the N- policy principle with rate ψ , the server returns to a busy state from vacation state.

When the operating unit is broken-down, it is located with probability coverage c. If the device is not effectively replaced, the reboot phase is conducted to continue operating the system. The fault is recovered, i.e., the failed machine is removed with parameter r. The reboot process follows an exponential distribution. The server is vulnerable to failure; the lifetime of the server is exponentially distributed with mean $1/\alpha$. The repair process follows a threshold recovery and is carried out following exponential distribution with mean $1/\beta$. In threshold recovery policy, the repairs of the failed units are initiated after the accumulation of repair jobs i.e., when the system size reaches a threshold level $'q'$.

3.2 Mathematical formulation of model

Consider thesystem states and respective probabilities two random processes ($I(\tau), \eta(\tau)$), which describe the mechanism entirely, to describe the state of the queueing system at any instant. The bivariate stochastic process $\chi(\tau)=\{I(\tau),\eta(\tau);\tau\geq 0\}$ is used to formulate Markov model, where

i) The server's status is signified as $I(\tau)$ and takes values 0,1, 2, 3, 4 and 5, and

ii) The total number of failed machines in the system $\eta(\tau)(\eta(\tau)=0,1,2,...,M)$.

Now we define system state probabilities by $\pi_{i,n}(\tau)$ where at time epoch $'\tau'$ there are n failed machines in the system and the server status is denoted as $I(\tau)=i, 0\leq i\leq 5$. For various states of the system, the probabilities of transient behavior are defined in the following manner:

(i) Vacation state of the server.

$$\pi_{0,n}(\tau)=\{I(\tau)=0,\eta(\tau);\tau\geq 0\}, 0\leq n\leq M-1$$

(ii) Server is under reboot during vacation state.

$$\pi_{1,n}(\tau)=\{I(\tau)=1,\eta(\tau);\tau\geq 0\}, 1\leq n\leq M-1$$

(iii) Normal busy state of the server.

$$\pi_{2,n}(\tau)=\{I(\tau)=2,\eta(\tau);\tau\geq 0\}, 0\leq n\leq M$$

(iv) During normal busy state, server is under reboot.

$$\pi_{3,n}(\tau)=\{I(\tau)=3,\eta(\tau);\tau\geq 0\}, 1\leq n\leq M-1$$

(v) When broken-down during normal busy state, server is under repair.

$$\pi_{4,n}(\tau)=\{I(\tau)=4,\eta(\tau);\tau\geq 0\}, 0\leq n\leq M-1$$

(vi) Server is under reboot during repair state.

$$\pi_{5,n}(\tau)=\{I(\tau)=5,\eta(\tau);\tau\geq 0\}, 1\leq n\leq M-1$$

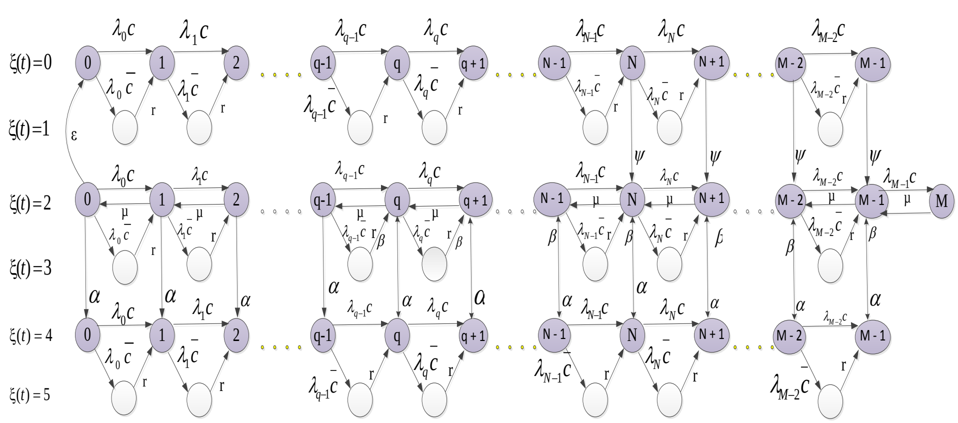

Figure 1: State transition flows of FTS with threshold recovery

4. GOVERNING EQUATIONS

To construct Markov model Chapman-Kolmogorov equations are generated using the flow conservation law for transition rate. Based on birth-death process for the different server states i.e. $I(\tau)=i, i=0,1,2,3,4,5.$, the governing equations for transient states, are constructed as follows:

i) **Vacation state of the server, when** $I(\tau)=0$.

$$\pi'_{0,0}(\tau)=\varepsilon\pi_{2,1}(\tau)-\lambda_{0,}\pi_{0,0}(\tau) \tag{1}$$

$$\pi'_{0,n}(\tau)=\lambda_{n-1}c\pi_{0,n-1}(\tau)+r\pi_{1,n}(\tau)-\lambda_n\pi_{0,n}(\tau);\ 1\le n\le N-1 \tag{2}$$

$$\pi'_{0,n}(\tau)=\lambda_{n-1}c\pi_{1,n-1}(\tau)+r\pi_{1,n}(\tau)-(\lambda_n+\psi)\pi_{0,n}(\tau);\ N\le n\le M-2 \tag{3}$$

$$\pi'_{0,M-1}(\tau)=\lambda_{M-2}c\pi_{0,M-2}(\tau)+r\pi_{1,M-1}(\tau)-\psi\pi_{0,M-1}(\tau) \tag{4}$$

ii) **The server is under reboot while being in vacation state, when** $I(\tau)=1$.

$$\pi'_{1,1}(\tau)=\lambda_0\bar{c}\pi_{0,0}(\tau)-r\pi_{1,1}(\tau) \tag{5}$$

$$\pi'_{1,n}(\tau)=\lambda_{n-1}\bar{c}\pi_{0,n-1}(\tau)-r\pi_{1,n}(\tau);\ 2\le n\le M-1 \tag{6}$$

iii) **Normal busy state of the server, when** $I(\tau)=2$.

$$\pi'_{2,0}(\tau)=\mu\pi_{2,1}(\tau)-(\lambda_0+\varepsilon+\alpha)\pi_{2,0}(\tau) \tag{7}$$

$$\pi'_{2,n}(\tau)=\mu\pi_{2,n+1}(\tau)+r\pi_{3,n-1}(\tau)-(\lambda_n+\mu+\alpha)\pi_{2,n}(\tau);\ 1\le n\le q-1 \tag{8}$$

$$\pi'_{2,n}(\tau)=\mu\pi_{2,n+1}(\tau)+r\pi_{3,n-1}(\tau)+\beta\pi_{4,n}(\tau)-(\lambda_n+\mu+\alpha)\pi_{2,n}(\tau);\ q\le n\le N-1 \tag{9}$$

$$\begin{aligned}\pi'_{2,n}(\tau)&=\mu\pi_{2,n+1}(\tau)+r\pi_{3,n}(\tau)+\beta\pi_{4,n}(\tau)+\psi\pi_{0,n}(\tau)\\&-(\lambda_n+\mu+\alpha)\pi_{2,n}(\tau);\quad N\le n\le M-1\end{aligned} \tag{10}$$

$$\pi^1_{2,M}(\tau)=\lambda_{M-1}c\pi_{2,M-1}(\tau)-\mu\pi_{2,M}(\tau) \tag{11}$$

iv) **Reboot mode from normal busy state of the server, when** $I(\tau)=3$.

$$\pi'_{3,1}(\tau)=\lambda_0\bar{c}\pi_{2,0}(\tau)-r\pi_{3,1}(\tau) \tag{12}$$

$$\pi'_{3,n}(\tau)=\lambda_{n-1}\bar{c}\pi_{2,n-1}(\tau)-r\pi_{3,n}(\tau);\ \text{for}\ 3\le n\le M-1 \tag{13}$$

v) **When server is under repair when failed normal busy state, when** $I(\tau)=4$.

$$\pi'_{4,0}(\tau)=\alpha\pi_{2,0}(\tau)-\lambda_0\pi_{4,0}(\tau) \tag{14}$$

$$\pi'_{4,n}(\tau)=\alpha\pi_{2,n}(\tau)+r\pi_{5,n+1}(\tau)+\lambda_{n-1}\pi_{4,n-1}(\tau)-\lambda_n\pi_{4,n}(\tau);\ 1\le n\le q-1 \tag{15}$$

$$\pi'_{4,n}(\tau)=\alpha\pi_{2,n}(\tau)+r\pi_{5,n+1}(\tau)+\lambda_{n-1}\pi_{4,n-1}(\tau)-(\lambda_n+\beta)\pi_{4,n}(\tau);\quad q\le n\le M-2 \tag{16}$$

$$\pi'_{4,M-1}(\tau)=\alpha\pi_{2,M-1}(\tau)+r\pi_{5,M-1}(\tau)+\lambda_{M-2}c\pi_{4,M-2}(\tau)-\beta\pi_{4,M-1}(\tau) \tag{17}$$

vi) **The server is under reboot state and reached there from repair state, when** $I(\tau)=5$.

$$\pi'_{5,1}(\tau)=\lambda_0\bar{c}\pi_{4,0}(\tau)-r\pi_{5,1}(\tau) \tag{18}$$

$$\pi'_{5,n}(\tau)=\lambda_{n-1}\bar{c}\pi_{4,n-1}(\tau)-r\pi_{5,n}(\tau);\quad 2\le n\le M-1 \tag{19}$$

5. THE MATHEMATICAL ANALYSIS

Spectral theory is used in this section to solve differential equations by using 'matrix form'. First, we take Laplace transforms and then placed the equations in block matrix. The following set of equations is obtained by Laplace transformation of (1) - (19).

i) **For** $I(\tau)=0$.

$$(s+\lambda_0)\pi^*_{0,0}(s)-\varepsilon\pi^*_{2,1}(s))=\pi_{0,0}(0) \tag{20}$$

$$(s+\lambda_n)\pi^*_{0,n}(s)-\lambda_{n-1}c\pi^*_{0,n-1}(s)-r\pi^*_{1,n}(s)=\pi_{0,n}(0);\, 1\le n\le N-1 \tag{21}$$

$$(\lambda_n+\psi+s)\pi^*_{0,n}(s)-\lambda_{n-1}c\,\pi^*_{1,n-1}(s)-r\pi^*_{1,n}(s)=\pi_{0,n}(0);\quad N\le n\le M-2 \tag{22}$$

$$(\psi+s)\,\pi^*_{0,M-1}(s)-\lambda_{M-2}c\,\pi^*_{0,M-2}(s)-r\pi^*_{1,M-1}(s)=\pi_{0,M-1}(0) \tag{23}$$

ii) **For** $I(\tau)=1$.

$$(s+r)\pi^*_{1,1}(s)-\lambda_0\bar{c}\,\pi^*_{0,0}(s)=\pi_{1,1}(0) \tag{24}$$

$$(s+r)\pi^*_{1,n}(s)-\lambda_{n-1}\bar{c}\,\pi^*_{0,n-1}(s)=\pi_{1,n}(0);\;\; 2\le n\le M \tag{25}$$

iii) **For** $I(\tau)=2$.

$$(s+\lambda_0+\varepsilon+\alpha)\pi^*_{2,0}(s)-\mu\,\pi^*_{2,1}(s)=\pi_{2,0}(0) \tag{26}$$

$$(s+\lambda_n+\mu+\alpha)\pi^*_{2,n}(s)-\mu\,\pi^*_{2,n+1}(s)+r\pi^*_{3,n-1}\,(s)=\pi_{2,n}(0);\;\; 1\le n\le q-1 \tag{27}$$

$$(s+\lambda_n+\mu+\alpha)\pi_{2,n}(s)-\mu\pi_{2,n+1}(s)-r\pi_{3,n-1}(s)-\beta\pi_{4,n}(s)=\pi_{2,n}(0);\;\; q\le n\le N-1 \tag{28}$$

$$(s+\lambda_n+\mu+\alpha)\pi^*_{2,n}(s)-\mu\,\pi^*_{2,n+1}(s)-r\,\pi^*_{3,n}(t)-\beta\,\pi^*_{4,n}(t)-\psi\,\pi^*_{0,n}(s)=\pi_{2,n}(t);\qquad N\le n\le M-1 \tag{29}$$

$$\mu\,\pi^*_{2,M}(s)-\lambda_{M-1}c\,\pi^*_{2,M-1}(s)=\pi_{2,M}(0) \tag{30}$$

iv) **For** $I(\tau)=3$.

$$(s+r)\pi^*_{3,1}(s)-\lambda_0\bar{c}\,\pi^*_{2,0}(s)=\pi_{3,1}(0) \tag{31}$$

$$(s+r)\pi^*_{3,n}+\lambda_{n-1}\bar{c}\,\pi^*_{2,n-1}=\pi_{3,n}(0);\;\; 3\le n\le M-1 \tag{32}$$

v) **For** $I(\tau)=4$.

$$\lambda_1\pi^*_{4,0}(s)-\alpha\,\pi^*_{2,0}(s)=\pi_{4,0}(0) \tag{33}$$

$$\lambda_{n-1}\,\pi^*_{4,n-1}(s)-\lambda_n\,\pi^*_{4,n}(s)-\alpha\,\pi^*_{2,n}(s)-r\,\pi^*_{5,n+1}(s)=\pi_{4,n}(0);\;\; 1\le n\le q-1 \tag{34}$$

$$(s+\lambda_n+\beta)\,\pi^*_{4,n}(s)-\alpha\,\pi^*_{2,n}(s)-r\,\pi^*_{5,n+1}(s)-\lambda_{n-1}\,\pi^*_{4,n-1}(s)=\pi_{4,n}(0);\;\; q\le n\le M-2 \tag{35}$$

$$\beta\pi^*_{4,M-1}(s)-\alpha\,\pi^*_{2,M-1}(s)-r\,\pi^*_{5,M-1}(s)-\lambda_{M-2}c\,\pi^*_{4,M-2}(s)=\pi_{4,M-1}(0) \tag{36}$$

vi) **For** $I(\tau)=5$.

$$(s+r)\pi^*_{5,1}(s)-\lambda_0\bar{c}\,\pi^*_{4,0}(s)=\pi_{5,1}(0) \tag{37}$$

$$(s+r)\pi^*_{5,n}-\lambda_{n-1}\bar{c}\,\pi^*_{4,n-1}=\pi_{5,n}(0);\;\; 2\le n\le M-1 \tag{38}$$

The above-mentioned Equations (20)-(39) can be written in matrix form as

$$Y(s)Z*(s)=Z(0) \tag{39}$$

where,
$$Y(s)=\begin{pmatrix} A & 0 & E & 0 & 0 & 0 \\ B & G & 0 & 0 & 0 & 0 \\ 0 & -G & D & 0 & 0 & 0 \\ C & 0 & F & G & H & G \\ 0 & 0 & I & 0 & J & 0 \\ 0 & 0 & 0 & 0 & F & -G \end{pmatrix}_{(6M-2)\times(6M-2)} \quad (40)$$

We denote the unit matrix of order M by I_M . The all-sub-matrices A,B,C,D,E,F,G,H,I,J are square matrices of order M and are provided by

$$A=\begin{bmatrix} -(\lambda_0 c+\lambda_0 \bar{c}) & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_0 c & -(\lambda_1 c+\lambda_1 \bar{c}) & 0 & \ldots & \ldots & \ldots & 0 \\ 0 & \lambda_1 c & -(\lambda_2 c+\lambda_2 \bar{c}) & 0 & \ldots & \ldots & 0 \\ : & : & \lambda_2 c & -(\lambda_3 c+\lambda_3 \bar{c}) & 0 & \ldots & 0 \\ : & : & : & \lambda_3 c & -(\lambda_4 c+\lambda_4 \bar{c}) & 0 & 0 \\ 0 & : & : & 0 & \lambda_4 c & -(\lambda_5 c+\lambda_5 \bar{c}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_5 c & -\Psi \end{bmatrix}_{(M)\times(M)} \quad (41)$$

$$B=\begin{bmatrix} \lambda_0 \bar{c} & 0 & \ldots & \ldots & \ldots & 0 \\ 0 & \lambda_1 \bar{c} & 0 & \ldots & \ldots & 0 \\ : & 0 & \lambda_2 \bar{c} & 0 & \ldots & 0 \\ : & : & : & \lambda_3 \bar{c} & 0 & 0 \\ : & : & : & 0 & \lambda_4 \bar{c} & 0 \\ 0 & : & 0 & 0 & 0 & \lambda_5 \bar{c} \end{bmatrix}_{(M)\times(M)} \quad ; \quad C=\psi I_M \quad (42)$$

$$D=\begin{bmatrix} -(\mu+\alpha+\lambda_0 c+\lambda_0 \bar{c}) & \mu & 0 & \ldots.. & \ldots.. & \ldots.. & \ldots.. & \ldots.. & 0 \\ \lambda_0 c & -(\mu+\alpha+\lambda_1 c+\lambda_1 \bar{c}) & \mu & 0 & \ldots.. & \ldots.. & \ldots.. & \ldots.. & 0 \\ 0 & \lambda_1 c & -(\mu+\alpha+\lambda_2 c+\lambda_2 \bar{c}) & \mu & 0 & \ldots.. & \ldots.. & 0 & 0 \\ : & : & \lambda_2 c & -(\mu+\alpha+\lambda_3 c+\lambda_3 \bar{c}) & \mu & 0 & \ldots.. & 0 & 0 \\ : & : & : & \lambda_3 c & -(\mu+\alpha+\lambda_4 c+\lambda_4 \bar{c}) & \mu & \ldots.. & 0 & 0 \\ : & : & : & 0 & \lambda_4 c & -(\mu+\alpha+\lambda_5 c+\lambda_5 \bar{c}) & & \mu & 0 \\ : & : & : & 0 & 0 & \lambda_5 c & & -\lambda_5 c & \mu \\ 0 & 0 & 0 & 0 & 0 & 0 & & \lambda_6 c & 0 \end{bmatrix}_{(M)\times(M)}$$

$$E=\begin{bmatrix} \varepsilon & 0 \\ 0 & 0 \end{bmatrix}_{(M)\times(M)} \qquad F=\bar{c}\,diag[\lambda_0,\lambda_1,\lambda_2,\lambda_3,\ldots\lambda_M];\ G=rI_M;\ \ H=\beta I_M;\ I=\alpha I_M$$

$$J = \begin{bmatrix} -(\lambda_0 c + \lambda_0 \bar{c}) & 0 & .... & .... & .... & 0 \\ \lambda_0 c & -(\lambda_1 c + \lambda_1 \bar{c}) & 0 & .... & .... & : \\ 0 & \lambda_1 c & -(\lambda_2 c + \lambda_2 \bar{c} + \beta) & 0 & .... & : \\ : & 0 & \lambda_2 c & -(\lambda_3 c + \lambda_3 \bar{c} + \beta) & 0 & : \\ : & : & 0 & \lambda_3 c & -(\lambda_4 c + \lambda_4 \bar{c} + \beta) & 0 \\ : & : & : & 0 & \lambda_4 c & -(\lambda_5 c + \lambda_5 \bar{c} + \beta) \\ 0 & .... & .... & .... & 0 & \lambda_5 c \end{bmatrix}_{(M)\times(M)}$$

In the partitioned form, the unknown vector $X^*(u)$ can be represented as

$$X^*(u) = [X^*_{0,m}(u), X^*_{1,m}(u), X^*_{2,m}(u), X^*_{3,m}(u), X^*_{4,m}(u), X^*_{5,m}(u)]^T \tag{43}$$

where $X^*_{0,m}(u) = [X^*_{0,0}(u), X^*_{0,1}(u), \ldots, X^*_{0,q}(u), X^*_{0,N-1}(u), X^*_{0,N}(u), .., X^*_{0,M-1}(u)]^T$;

$X^*_{1,m}(u) = [X^*_{1,1}(u), \ldots.., X^*_{1,M-1}(u)]^T$;

$X^*_{2,m}(u) = [X^*_{2,0}(u), X^*_{2,1}(u), \ldots, X^*_{2,q}(u), X^*_{2,N-1}(u), X^*_{2,N}(u), .., X^*_{2,M}(u)]^T$;

$X^*_{3,m}(u) = [X^*_{3,1}(u), \ldots.., X^*_{3,M-1}(u)]^T$;

$X^*_{4,m}(u) = [X^*_{4,0}(u), X^*_{4,2}(u), \ldots, X^*_{4,q}(u), X^*_{4,N-1}(u), X^*_{4,N}(u), .., X^*_{4,M-1}(u)]^T$;

$X^*_{5,m}(u) = [X^*_{5,1}(u), \ldots.., X^*_{5,M-1}(u)]^T$;

Here, initial vector $X(0)$ is defined as

$$X(0) = [1, 0, 0, ..., 0, 0, 0, ..., 0, 0, ..., 0, 0, 0]_{(6M-2)\times 1}$$

Now, we introduce Crammer rule to the matrix $Y(s)$ to obtain the transient state probabilities.

$$X^*_{i,m}(u),\ (i = 0,1,...,5; m = 0,1,...,M).$$

$$X^*_{i,m}(u) = \frac{\det[Y_{k+1}(u)]}{\det[Y(u)]},\ \text{where } (k = (i-1)(M+1) + m; i = 0,1,2,3,4,5; m = 0,1,...,M) \tag{44}$$

where, by modifying j^{th} column of det [Y(u)] with the elements of initial vector $X(0)$, $Y_{k+1}(u)$ is acquired.

To solve the equation (39), we simply continue to determine the characteristic root of the matrix $Y(u)$. It is figured out that one of the roots is $r_0 = 0$. Let $u = (-d)$, so we'll have

$$Y(-r) = Y(-r\,I) \tag{45}$$

Now Equation (39) converts into

$$Y(-r)X^*(u) = (Y - r\,I)Y^*(u) = X(0) \tag{46}$$

We noticed that e and n real roots in pairs of complex roots, respectively

We represent real roots by: $r_1, r_2, ..., r_r$ and $(r_{r+1}, \bar{r}_{r+1}), (r_{r+2}, \bar{r}_{r+2}), ..., (r_{r+n}, \bar{r}_{r+n})$, respectively.

Now, we have

$$|Y(u)| = u\left[\prod_{j=1}^{e}(u + r_j)\right]\left[\prod_{j=1}^{n}(u + r_{e+j})(u + \bar{r}_{e+j})\right] \tag{47}$$

Equations (37) and (41) yield

$$X^*_{i,m}(u) = \frac{|Y(u)|}{u\left[\prod_{j=1}^{e}(u + r_j)\right]\left[\prod_{j=1}^{n}(u + r_{e+j})(u + \bar{r}_{e+j})\right]}, \tag{48}$$

Equation (42) in partial fraction form can be written as follows

$$X_{i,m}^{*}(u)=\frac{\mathbf{a_0}}{u}+\frac{\mathbf{a_1}}{u+r_1}+\ldots+\frac{\mathbf{a}_r}{u+r_r}+\frac{\mathbf{b}_r u+\mathbf{c}_r}{(u+r_{r+1})(u+\bar{r}_{r+1})}+\ldots+\frac{\mathbf{b}_n u+\mathbf{c}_n}{(u+r_{r+n})(u+\bar{r}_{r+n})} \tag{49}$$

a_0 and a_q $(q=1,2,\ldots,e)$ are real numbers here, determined as:

$$\mathrm{a}_0=\frac{\left|Y_{j+1}(\mathrm{u})\right|}{\left(\prod_{\mathrm{j}=1}^{e} r_{\mathrm{j}}\right)\left(\prod_{\mathrm{j}=1}^{n} r_{1+\mathrm{j}}\,\bar{r}_{1+\mathrm{j}}\right)} \tag{50}$$

$$\mathrm{a}_{\mathrm{q}}=\frac{\left|\mathrm{Q}_{\mathrm{j}+1}(-\mathrm{r}_{\mathrm{p}})\right|}{(-\mathrm{d}_{\mathrm{q}})\left[\prod_{\substack{\mathrm{j}=1\\ \mathrm{j}\neq\mathrm{q}}}^{e}(\mathrm{r}_{\mathrm{j}}-\mathrm{r}_{\mathrm{q}})\right]\left[\prod_{\substack{\mathrm{j}=1\\ \mathrm{j}\neq\mathrm{q}}}^{\mathrm{n}}(\mathrm{r}_{e+j}-\mathrm{r}_{\mathrm{q}})(\bar{\mathrm{r}}_{e+j}-\mathrm{r}_{\mathrm{q}})\right]},\ \mathrm{q}=1,2,\ldots,\mathrm{r}. \tag{51}$$

A combination of real part u_p and imaginary part v_p is a combination of complex characteristic root d_{e+p}. Then,

$$\mathrm{b}_{\mathrm{p}}(-r_{e+\mathbf{p}})+\mathrm{c}_{\mathrm{p}}=\frac{\left|Y_j(-r_{e+p})\right|}{(-r_{r+p})\left[\prod_{\substack{j=1\\ j\neq p}}^{r}(r_j-r_{e+p})\right]\left[\prod_{\substack{j=1\\ j\neq p}}^{m}(r_{e+j}-r_{e+p})(\bar{r}_{e+j}-r_{e+p})\right]};\ p=1,2,\ldots,n \tag{52}$$

By taking Equation's inverse Laplace transform (48), we obtain

$$Q_{i,m}(\mathrm{t})=\mathrm{a}_0+\sum_{\mathrm{q}=1}^{r}\mathrm{a}_q\mathrm{e}^{-\mathrm{d}_q\mathrm{t}}+\sum_{\mathrm{p}=1}^{n}\left[\mathrm{b}_{\mathrm{p}}\mathrm{e}^{-\mathrm{u}_{\mathrm{p}}\mathrm{t}}\cos(\mathrm{v}_{\mathrm{p}}\mathrm{t})+\frac{\mathrm{c}_{\mathrm{p}}-\mathrm{b}_{\mathrm{p}}\mathrm{u}_{\mathrm{p}}}{\mathrm{v}_{\mathrm{p}}}\mathrm{e}^{-\mathrm{u}_{\mathrm{p}}\mathrm{t}}\sin(\mathrm{v}_{\mathrm{p}}\mathrm{t})\right]; \tag{53}$$

where, $(j=(i-1)(M+1)+m; i=0,1,\ldots,5; m=0,1,\ldots,M)$ and the real numbers are $a_0, a_p, d_q, b_p, c_p, u_p$ and v_p.

6. PERFORMANCE MEASURES AND SENSITIVITY ANALYSIS

The performance characteristics of FTS with reboot and standby can be examined in terms of performance metrics. For the transient behavior of the system, now we establish the performance of the system as follows:

6.1 Queueing indices

The following performance measures are built up in various scenarios to analyses the real-time FTS

i) The mean number of failed units in the system

$$EN(\tau)=n.[\sum_{n=0}^{M}\pi_{2,n}(\tau)+\sum_{n=0}^{M-1}\sum_{i=0,4}\pi_{i,n}(\tau)+\sum_{n=1}^{M-1}\sum_{i=3,5}\pi_{i,n}(\tau)] \tag{54}$$

ii) Machine availability

$$MA(\tau)=1-\frac{EN(\tau)}{M} \tag{55}$$

iii) Failure Frequency

$$f(\tau)=\alpha\sum_{n=1}^{M}\pi_{4,n}(\tau) \tag{56}$$

iv) System throughput is given by

$$TP(\tau)=\sum_{n=1}^{M}\mu\pi_{2,n}(\tau) \tag{57}$$

6.2 Transient system states

The transient probabilities related to various server's status that could be in normal busy state $(P_B(\tau))$, reboot state $(P_R(\tau))$,under repair $(P_{BD}(\tau))$, and on vacation ($P_V(\tau)$) are derived as follows:
Thus,

i) The system is in reboot state at time τ

$$P_R(\tau)=\sum_{n=1}^{M-1}\pi_{1,n}(\tau)+\sum_{n=1}^{M-1}\pi_{3,n}(\tau)+\sum_{n=1}^{M-1}\pi_{5,n}(\tau) \quad (58)$$

ii) The system is in failed state at time τ

$$P_{BD}(\tau)=\sum_{n=0}^{M-1}\pi_{4,n}(\tau) \quad (59)$$

iii) The system is in normal busy state at time τ

$$P_B(\tau)=\sum_{n=0}^{M}\pi_{2,n}(\tau) \quad (60)$$

iv) The system is in vacation state at time τ

$$P_V(\tau)=\sum_{n=0}^{M-1}\pi_{0,n}(\tau) \quad (61)$$

6.3 Reliability function and MTTF

(i) The $R_Y(\tau)$ of the system and MTTF are obtained using

$$R_Y(\tau)=1-\pi_{2,M}(\tau) \quad (62)$$

(ii) Mean time to system failure (MTTF) is obtained

$$MTTF=\int_0^{\infty}R_Y(\tau)\,d\tau=\lim_{s\to 0}[1/s-\pi_{2,M}(\tau)] \quad (63)$$

6.4 System cost

The total cost involved by the system is useful to be determined, so that industrial engineers can gain insight into the enhancement of the system's future design. The overall estimated cost is the representation of the different cost elements corresponding to the various operations of the system under consideration.To anticipate the costs involved in the system's operation, the estimated cost function is formulated which is the composition of the cost components associated with the machining system. The following are the cost components incurred in various activities:

C_R :Cost/unit time under reboot

C_{BD} : Cost/unit time in broken-down state

C_μ : Cost/unit time incurred on the repair of each broken down machine

C_{EN} : Holding cost/unit time of each failed unit

C_V : Cost per unit time spend on the vacation state of the repairman

C_B :Cost per unit timeon the repairman in normal busy state

The cost function is set up as follows by taking into consideration the cost components, queueing indices and system state probabilities:

$$TC(\tau)=C_{EN}EN(\tau)+C_BP_B(\tau)+C_{BD}P_{BD}(\tau)+C_RP_R(\tau)+C_VP_V(\tau)+\mu C_\mu \quad (64)$$

7. NUMERICAL RESULTS AND SENSITIVITY ANALYSIS

The numerical outcomes for various performance metrics are examined, to demonstrate the impact of system parameters. The system performance is easy to understand with the help of numerical results shown in the graphs (2-4) and tables (2-6). For numerical simulation, MATLAB is used so as to evaluate the system performance. The default parameters are taken as: $\mu=4; r=3; \lambda=0.1; \psi=0.5; \alpha=0.5; \varepsilon=0.5; c=0.5; \beta=1$ for the computational purpose. The transient behavior is investigated by evaluating the performance results numerically and shown in Tables 2-4. Numerical results for various performance metrics are obtained by varying values of λ, μ, α and β. The influence of the system descriptors is now being investigated as follows:

(i) **Effect of failure rate of operating units** (λ)

Table 2 demonstrates the impact of λ on different performance metrics i.e. queue length, server probability to be in various system states, machine availability and overall system cost. From Table 2 and Figure 2(a) it is observed that $EN(\tau)$ increases as λ increases. It is noticeable that with an increase in λ, there is an enhancement in the counting of broken-down machines. It is clearly observed from Table 2 and Figure 4(a) that the availability of the machine seems to decrease with an increment in λ. The total system cost increases with time as failure rate of operating units increases. Also, from Figure 3(a) it is clear that with the increase in λ, there is decrement in system reliability.

(ii) **Effect on various performance metrics of repair rate of failed machines** (μ)

The effect of repair rate (μ) on various performance indices viz.queue length, probability to be in various system states, machine availability and overall system cost is depicted in Table 3. Moreover, the system cost shows an increasing trend with the increase in time and also there is increment with respect to μ. As predicted and demonstrated in several real-life circumstances, it is observed from Figure 2(b) and Table 3, that as there is increment in the value of repair rate (μ), $EN(\tau)$ decreases. On the other hand, the availability of the system $MA(\tau)$ indicates an increasing trend towards an increase in μ. From Figure 3(b), it is noticed that $R_Y(\tau)$ shows an increment for the greater values of repair rate. In many complex systems, the repair rate may indeed have a significant role in achieving the pre-determined reliability.

Table 2: Variation in performance measures for various values of failure rate (λ)

	τ	$EN(\tau)$	$P_B(\tau)$	$P_{BD}(\tau)$	$P_V(\tau)$	$P_R(\tau)$	$MA(\tau)$	$TC(\tau)$
$\lambda = 0.6$	0	0	1	0	0	0	1	420
	1	1.7054	0.2945	0.0828	0.0798	0.6426	0.7868	347.9397
	2	2.4113	0.2323	0.0776	0.0737	0.6481	0.6986	405.8233
	3	2.9695	0.2119	0.0762	0.0771	0.6219	0.6288	456.2954
$\lambda = 0.8$	0	0	1	0	0	0	1	420
	1	1.7232	0.2992	0.0789	0.1001	0.6417	0.7846	345.0546
	2	2.4462	0.2731	0.0735	0.092	0.6462	0.6942	405.1254
	3	3.0189	0.2826	0.0718	0.0965	0.6191	0.6226	456.7476
$\lambda = 1$	0	0	1	0	0	0	1	420
	1	1.7393	0.2827	0.0753	0.1179	0.641	0.7826	342.5952
	2	2.4774	0.2986	0.0698	0.1082	0.6446	0.6903	404.5606
	3	3.0628	0.3225	0.0679	0.1138	0.6165	0.6172	457.1588

Table 3: Variation in performance measures for various values of service rate (μ)

	τ	$EN(\tau)$	$P_B(\tau)$	$P_{BD}(\tau)$	$P_V(\tau)$	$P_R(\tau)$	$MA(\tau)$	$TC(\tau)$
μ =1.5	0	0	1	0	0	0	1	397.5
	1	1.7799	0.185	0.0755	0.1006	0.639	0.7775	328.4992
	2	2.5539	0.1971	0.0689	0.0932	0.6406	0.6808	393.4827
	3	3.1781	0.2234	0.0662	0.0987	0.6098	0.6027	450.0192
μ = 3	0	0	1	0	0	0	1	465
	1	1.6289	0.1695	0.085	0.0991	0.6464	0.7964	380.0933
	2	2.2733	0.1725	0.0822	0.09	0.6551	0.7158	432.4627
	3	2.7669	0.19	0.0828	0.0929	0.6331	0.6541	476.4077
μ =4.5	0	0	1	0	0	0	1	532.5
	1	1.5247	0.158	0.0926	0.0979	0.6515	0.8094	436.4337
	2	2.0941	0.1549	0.0933	0.0876	0.6641	0.7382	481.2867
	3	2.5148	0.1664	0.0973	0.089	0.6463	0.6856	517.9524

Table 4: Variation in performance measures for various values of failure rate (α)

	τ	$EN(\tau)$	$P_B(\tau)$	$P_{BD}(\tau)$	$P_V(\tau)$	$P_R(\tau)$	$MA(\tau)$	$TC(\tau)$
α = 0.5	0	0	1	0	0	0	1	397.5
	1	2.6374	0.1267	0.0397	0.0159	0.8157	0.6703	381.2063
	2	3.3203	0.1293	0.0464	0.0135	0.7882	0.585	434.9727
	3	3.7177	0.1413	0.0527	0.0127	0.7293	0.5353	466.8896
α = 0.7	0	0	1	0	0	0	1	397.5
	1	2.6455	0.1149	0.0522	0.0155	0.8152	0.6693	378.6588
	2	3.3177	0.1155	0.0599	0.0131	0.7863	0.5853	430.8412
	3	3.6876	0.1241	0.0672	0.0123	0.7252	0.539	459.3758
α = 0.9	0	0	1	0	0	0	1	397.5
	1	2.653	0.1047	0.063	0.0151	0.8148	0.6684	376.4672
	2	3.3163	0.1038	0.0713	0.0128	0.7847	0.5855	427.4275
	3	3.6641	0.1101	0.0791	0.0121	0.7218	0.542	453.3055

(iii) **Effect of server failure rate** (α)

The influence of α on $EN(\tau)$, system state probabilities of FTS, machine availability and overall system cost is illustrated in Table 4. It is observed from Table 4 and Figure 2(c) that with the increase in server failure rate, an increasing trend is clearly observed in $EN(\tau)$. The probability of server being in broken down state increases with the increase in the failure rate of the server. In several machining systems, it is discovered that the availability of machines decreases as the value of α increases. It is clear from Table 4 and Fig. 4(c) that the machine availability shows decrement as the rate of failure increases. The increase in server's breakdown rate also affects the reliability $R_Y(\tau)$ which seems to decrease as seen in Figure 3(c). There are significant changes in the probabilities of the server to be in reboot state, busy state and vacation state with the increment in α .

(iv) **Impact on various performance metrics with time** (τ)

From Tables 2-4, it is examined that the server's probability to be in busy state increases as time (τ) increases. However, it is evident that the server's probabilities to be in broken-down state, vacation state and reboot state show decreasing trends as time (τ) increases. It is observed in Figure. 2(a)-2(c) that $EN(\tau)$ indicates an increasing trend with growing time (τ). From Figure 4(a)-4(c) it is clearly noticed that with an increment in time (τ), there is decrement in $MA(\tau)$. The trend of $R_Y(\tau)$ with respect to time (τ) can be seen in Figure 3(a)-3(c). From these figures it is discovered that the reliability $R_Y(\tau)$ decreases as time τ increases.

(v) Effect on *MTTF* of various system parameters

Numerical outcomes are used to identify the impact of different parameters of the system on the *MTTF*. The results obtained for *MTTF* are presented in Tables 5(a)-5(f) for various values of system parameters. *MTTF* increases with the increment in the values of μ and β. But decline is observed in the values of *MTTF* when there is an increment in the values of failure rates, standby rate, breakdown rate, as well as in reboot/ vacation rate.

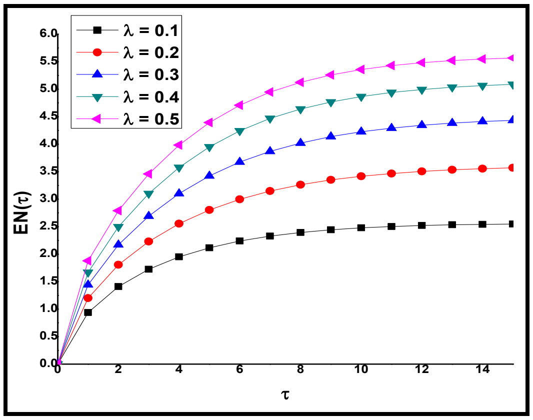

Figure 2(a)

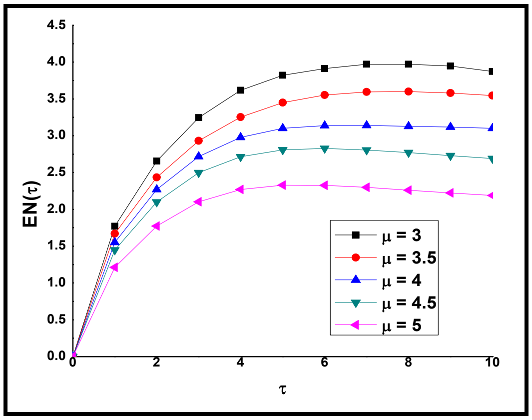

Figure 2(b)

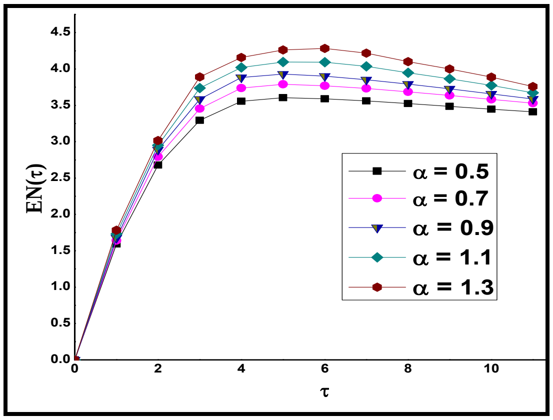

Figure 2(c)

Fig. 2.$EN(\tau)$for changing values of(a) λ ; (b) μ and (c) α

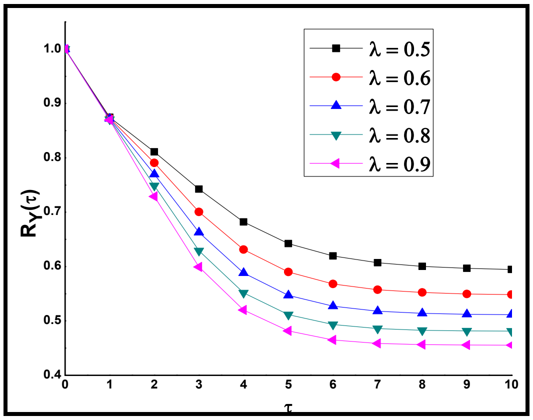

Figure 3(a)

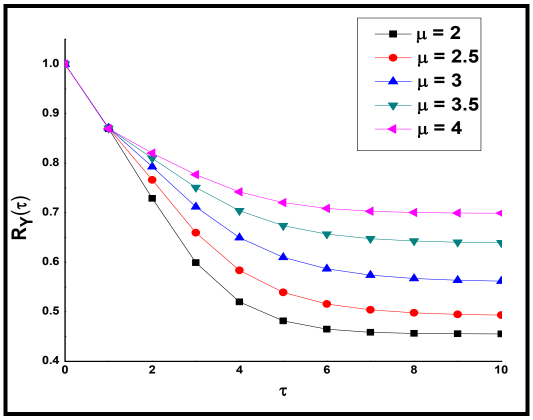

Figure 3(b)

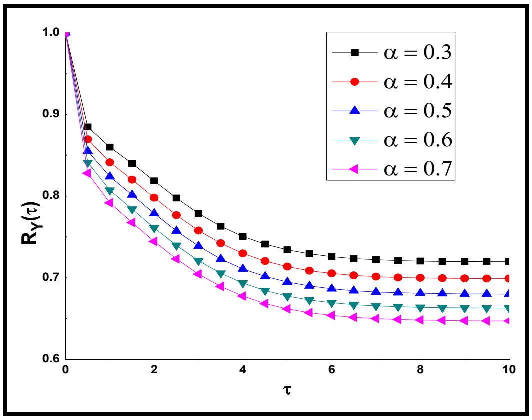

Figure 3(c)

Fig. 3. $R_Y(\tau)$ **for changing values of(a)** λ **; (b)** μ **and (c)** α

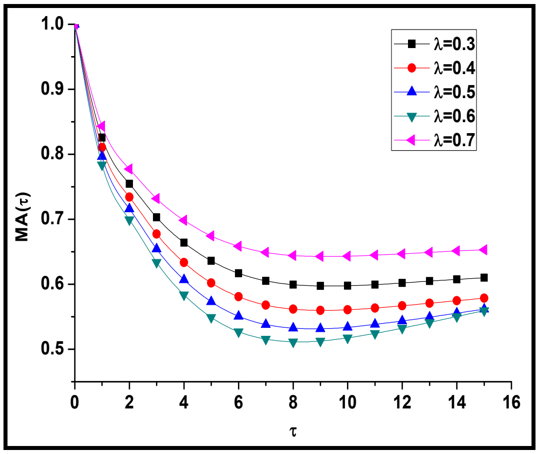

Figure 4(a)

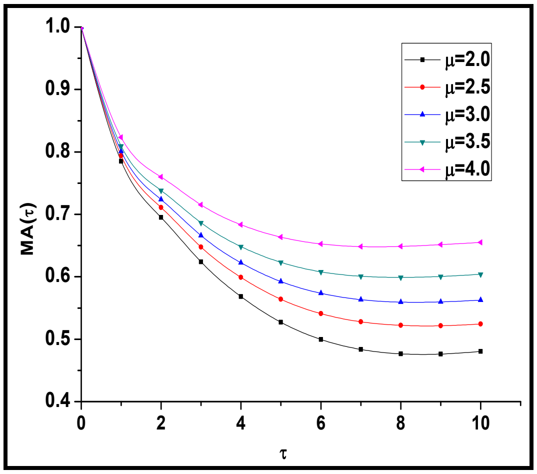

Figure 4(b)

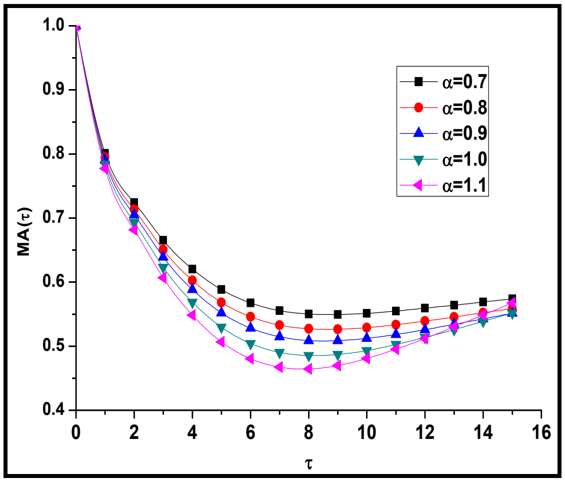

Figure 4(c)

Figure 4: $MA(\tau)$ for changing values of(a) λ ; (b) μ and (c) α

Table 5: MTTF for different values of various parameters: (a) λ, μ **(b)** λ, ε **(c)** λ, β **(d)** λ, r **(e)** λ, α **(f)** λ, γ

Table 5(a)

λ	μ = 2	μ = 3	μ = 4	μ = 5	μ = 6
0.1	51.2654	60.2565	69.831	77.9943	85.7767
0.2	42.6598	48.3265	54.9963	59.9997	65.2233
0.3	37.3265	41.2658	46.7878	50.0036	54.6464
0.4	34.2658	37.2114	41.3331	45.0398	48.7439
0.5	31.2655	35.8197	38.4944	41.1212	44.3594
0.6	30.2564	33.8974	36.0069	39.3394	42.5551

Table 5(b)

λ	ε = 0.6	ε = 0.7	ε = 0.8	ε = 0.9	ε = 1
0.5	40.0268	40.0069	39.9885	39.9712	39.9551
0.6	30.4819	30.4734	30.4654	30.4579	30.4508
0.7	25.5112	25.5084	25.5057	25.5031	25.5007
0.8	22.5579	22.5585	22.5591	22.5597	22.5602
0.9	20.6389	20.6419	20.6448	20.6476	20.6503
1	19.3098	19.3146	19.3191	19.3235	19.3277

Table 5(c)					
λ	$\beta = 2$	$\beta = 3$	$\beta = 4$	$\beta = 5$	$\beta = 6$
0.5	45.0131	48.5444	52.6162	55.8865	60.7373
0.6	36.5548	37.7878	39.7253	40.7037	44.7058
0.7	28.772	29.8009	31.0039	32.9759	35.3098
0.8	25.6183	26.8823	28.6739	30.6327	31.586
0.9	23.289	23.9779	24.8097	26.8807	27.6072
1	22.2112	22.7749	23.8124	24.8273	25.9883

Table 6: MTTF for different values of various parameters:
(a) λ, r **(b)** λ, α **(c)** λ, γ

Table 6(a)					
λ	$r = 1.5$	$r = 2$	$r = 2.5$	$r = 3$	$r = 3.5$
0.5	90.0215	76.4685	64.0298	54.4748	51.3137
0.6	75.4698	63.4414	53.5009	45.9856	42.6454
0.7	67.3501	56.3948	47.8156	40.4044	38.0098
0.8	62.3059	52.6664	43.1591	37.0001	35.8228
0.9	59.326	49.8524	41.9015	35.9898	33.6998
1	57.4509	46.0125	40.9719	35.3021	32.8895

Table 6(b)					
λ	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$
0.5	50.2154	45.0439	41.0319	38.5546	36.9969
0.6	39.4865	36.1987	32.4309	31.6565	29.777
0.7	33.9474	30.7319	28.3841	26.9339	25.8007
0.8	29.6514	27.5379	26.1819	24.548	22.01
0.9	27.3136	25.913	24.2249	22.34	21.1767
1	26.3089	24.0038	23.3111	21.0904	20.1909

Table 6(c)					
λ	$\gamma = 0.3$	$\gamma = 0.4$	$\gamma = 0.5$	$\gamma = 0.6$	$\gamma = 0.7$
0.5	37.886	37.6751	36.8476	36.2105	35.8943
0.6	33.731	33.0571	32.7291	32.1039	31.4685
0.7	27.9927	27.1283	26.4987	26.4961	26.0039
0.8	23.2419	23.0019	22.4982	22.1119	21.5984
0.9	19.771	19.2781	18.649	18.2091	17.816
1	17.7391	17.1984	16.6771	16.2323	15.9439

8. CONCLUSIONS AND DISCUSSION

In this study, we examined the queueing and reliability characteristics of FTS with reboot, vacation and threshold recovery. The explicit metrics for $R_Y(\tau)$ and MTTF are provided. The analytical results obtained in this study are based on simple mathematical tools and provide closed-form expressions can be easily used for computational purpose. The implementation of the concepts of working breakdown and threshold recovery will be helpful in minimizing the overall system cost as well as will provide additional gains for improving the system's capacity and availability. The principle of N-policy and threshold based recovery would play an eminent role in attaining the desired objective at optimum cost for the maintenance and the reboot process for the concerned failure-prone FTS. The sensitivity analysis carried out shows that the reliability and MTTF of the FTS can be enhanced by managing the appropriate parameters. The assessment of reliability indices of the FTS by including the realistic features may be utilized for capacity enhancement and up-gradation of fault tolerance, it has many uses in real time FTS including telecommunications, production system, power plants, service centers, etc. By taking into account the idea of working vacation, the concept of vacations may be further modified. The idea of the admission control policy can also be examined a future work by including the F-policy concept.

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