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HYBRID FAULT-TOLERANT REDUNDANCY FOR r-to-l-out-of-n:G SYSTEMS WITH OPTIMAL POLICY

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ABSTRACT

Now-a-days, industries and enterprises have become more dependent on the advance technology to perform technical and business activities as such there are high expectations for “always-on” of computers and electronic devices. Fault tolerance technique is very useful for the smooth functioning of such systems. Here we investigate a reliability model based on r-to-l-out-of-n: G architecture having provision of hybrid fault-tolerant redundancy (H.F.T.R.) and sparing. In this paper, the optimal strategy of hybrid redundancy has been applied to make system reliable. The proposed method allows a simple way for calculating the redundant parts by minimizing the mean overall system expenditure.

Keywords: Optimal policy, r-to-l-out-of-n: G, Fault-tolerant redundancy, Spare components, Reliability.

1. INTRODUCTION

A lot of researches on reliability modeling for redundant repairable systems to keep system fault tolerant, has been done. In reliability literature, k-out-of-: G i.e.: G system is the most eminent model firstly developed by Birnbaum et al. (1961). This system can be defined in operation mode as a process associated with n components which performs if or more of the components are in working process. This type system may be applied in many military and industrial systems. A is considered as no less than and not more than l units out of total n units are required for the operations of any system. In such system working units will be in between k and l, otherwise the system may go down.

Phillips (1980) interpreted that (k,n) architecture is the more reliable than any other coherent systems for attaining the ultra-reliable systems having independent and identically units with two modes of failures. Usha kumari and Krishna moorthy (2004) investigated max (N,T) policy to investigate(k, n) architecture for a system with repair facility under the optimal policy. For lifetime and repair time of components are considered to be exponential distributed (Exp-D). The study of reliability models with redundancy is done by Kim (2017). He obtained numerical results for various performance indices using matrix-analytic method. Ardkan et al. (2016) discussed a (k, n) architecture of a system with maintainability and redundancy strategies. They considered the redundancy strategies to determine the decision variables. They used analytic method based on integer programing problem (IPP) to obtained of the optimal solution of the corresponding problem. Recently, Mannai and Gasmi (2020) analyzed a system having n operating parts. The system will work if less than k units and more than l units will work in the system out of n units’ else system may go fail.

2. MODEL DESCRIPTION

(k, n) architecture is quite popular and widely used in many fault-tolerant systems (FTS).A lot of studies have been done in the field of (k, n) systems. Chiang and Chiang (1986) described the relayed communication via consecutive(k, n) architecture. The study of algorithm for computing the reliability of a (k, n) system was suggested by Wu and Chen (1994). Cui and Xie (2005) discussed a generalized form to find the reliability of this system. The reliability of the self-dual (k, n) systems was studied by Carreras et al.(2008). Zhang et al. (2007) proposed the repairman’s single vacation to analyze(k, n): G system. An extended investigation on the stochastic model for the availability and reliability of a repairable (k, n) system was carried by Moghaddass (2011 a, b).A system of n units that fails if at least (k, n) units fail is called (k, n): F system. Gokhdere (2016) established an improvement method for the reliability of consecutive k-out-of-n: F systems through computational approach. An improved disjoint products algorithm to evaluate the reliability of (k, n) system was introduced by Rushdi et al.(2018). Chang et. al. (2020) used universal generating function and linear programming to examine the reliability of (k, n) systems by sensing the potentially brittle behavior of the components.

The redundancy approach is being used to have a more reliable system with unreliable parts. The standby redundancy can be used to decrease the uncertainty of the system by increasing the average cost of the system. The optimization technique in reliability is the primary aims in advanced industries. The problem of optimum redundancy to maximize the reliability as well as to minimize the mean system cost was discussed earlier by Barlow et.al. (1963), Ray (1964), Siewiorek (1975), Tillman et.al. (1977), Ben-dov (1980). These researchers analyzed the problem of finding the optimal values of k which maximize the reliability of the system for given n.

Fault-tolerance is closely associated with maintaining system continuity via highly available digital systems and networks. Fault-tolerant digital systems were studied by Mathur (1971) and Ogus (1974). The problem of computing optimal number of redundant components which optimize the average cost rate was studied by Makagawa (1985). Hybrid redundancy can be defined as a redundant system having N-modular redundancy system with spares. Primarily Chu and Wah (1990) proposed and analyzed a fault redundancy neural network with hybrid redundancy. Fault-tolerant digital systems were studied by Mathur (1971) and Ogus (1974).

A $\left(k-l,n\right)$:G model can be applied in multiple fields. To find the reliability of $\left(k-l,n\right)$ systems, several researchers extended the early research works on*(k-n)* done by Jain and Gopal (1987), Rushdiand Dehlawi (1987), Rushdi (1987). Heidtmann (1984) provided a linear time algorithm for $\left(k-l,n\right)$:G system. Rushdi (1987 a,b), Bai et.al. (1991), Pham (1991) extended this research for analyzing the optimum redundancy with common-cause failures and determined the optimal system size with competing failures modes for a given value of k. Chen (1992) developed stochastic model to facilitate the transient analysis of reliability and availability. Pham and Phan (1992) obtained the average life of $\left(k-l,n\right)$system with non-identical weibull distributions functions. The optimal number of spares to minimize the total average cost of the system with hybrid fault-tolerant redundancy (H.F.T.R.) was also obtained by Pham (1992c).The $\left(k-l,n\right)$: model was investigated by Jain and Ghimire (1997) by considering random and common cause failures. Pham (1992b) determined the optimal system size of $\left(k-y-l,n\right)$: G systems by maximizing the mean profit. The m-consecutive $\left(k-l,n\right)$: F systems was discussed by Cui et al. (2015). Peng and Xiao (2018) obtained the reliability of consecutive (k, n) systems with two change points.

3. MODEL DESCRIPTION AND ANALYSIS

It is well known that $\left(k-l,n\right)$: G system has many practical applications such as in multiprocessor systems, computers, telecommunication networks, call centers and transportations systems. We extend the work of Pham (1992c) on (*r-n*) architecture by assuming a $\left(r-l,n\right)$:G system with hybrid fault tolerance redundancy (H.F.T.R). Here we consider a redundant system having modular redundancy (M.M.R.) and sparing. Our aim is to cut down the average cost of the system by finding the optimal number of spares.
We consider a $\left(r-l,n\right)$:G systems with the following assumption:

(i) The system is $\left(r-l,n\right)$: G with H.F.T.R.
(ii) Both the components and the systems can be in two states either operative or failed and $p\left(q\right)$ is the reliability (unreliability) of the components.
(iii) Components are assumed to be identical land independently distributed.
(iv) Sensing and switching are perfect.
(v) There are w spare modules with $2r-1=n$ active components.

The cost of the components (active or spare) and system failure are c_1 and c_2 respectively.)

4. THE ANALYSIS AND OPTIMIZATION

The reliability$Rel(w)$ of the considered system is given by

$$Rel(w) = \sum_{i=0}^{l-r+w} \binom{2r+w-1}{k+i} p^{r+i} q^{r+w-1-i} \tag{1}$$

Note that $Rel(w)$ is an increasing function of w. But system cost also increases with the increase in w.

Let us consider that $T(w)$ be the overall cost of theconcerned system. Now, the mean total cost of the proposed model is computedso that the optimal number of spare components(w)can be obtained by minimizing the mean total cost of the system. We construct the cost function by

$$\mathrm{E}[T(w)]=c_1(2r+w-1)+c_2[1-R(w)] \tag{2}$$

Thus, $$\Delta[T(w)]= E[T(w+1)]-E[\,(w)] \quad = c_1 - c_2\ [\,f(w)] \tag{3}$$

Where $$f(w) = \binom{2r+w-1}{l+w+1} p^{l+w-1} q^{2r-l-1} \tag{4}$$

Again, we calculate $$\Delta f(\mathrm{w}) = f(w+1) - f(w) = \binom{2r+w-1}{l+w+1} p^{l+w+1}\ q^{2r-l-1} \left(\frac{2r+w}{l+w+2}\ \ (p-1)\right) \tag{5}$$

For $$\Delta f(w) \geq 0,\ \ \frac{2r+w}{l+w+2} p \geq 1$$

$$\text{i.e.}\ \ w \leq \frac{2r-l-2}{q} - 2r$$

Define $$w_0 = \frac{2r-l-2}{q} - 2r \tag{6}$$

Then $\Delta f(w) \geq 0$ iff w$\leq$ w_0 and $\Delta f(w) < 0$ iff$w > w_0$.

Hence$f(w)$ is an increasing or decreasing function of w for $w \in [\,0, w_0\,]$

or$w \in [w_0\,, \infty\,)$, respectively.

For fixed values of q , r , l , d and c , the optimal value of w(say w^*) which minimizes the mean total cost of the system can be obtained in the same manner as discussed in Pham (1992). For our model, w^*will be zero (i. e., there is no need of spare part from optimum cost point of view) in the following two cases:

(i) If f $(w_0) < \frac{c_1}{c_2}$

(ii) If f $(w_0) \geq \frac{c_1}{c_2}$, f(0) $< \frac{c_1}{c_2}$

and $E\ [T(0)] \leq E[T(w_1)]$ where $w_1$is the smallest $w \in [w_0\,, \infty)$ satisfying $f(w) < \frac{c_1}{c_2}$.

The optimal value w^*becomes w when f$(w_0) \geq \frac{c_1}{c_2}$ and f(0) $< \frac{c_1}{c_2}$and $E[T(0)] > E[T(w_1)]$.

5. NUMERICAL RESULTS AND SENSITIVITY ANALYSIS

We compute the optimal number of spares w^*, the corresponding minimum mean cost E [T (w^*)] of the system and the resulting cost $\frac{\mathrm{E}\,[\mathrm{T}\,(w^*)]}{c_1}$ for some specific values of $\frac{c_1}{c_2}$ with distinct probability p. It is clear from Tables 1- 3 that for increasing $\frac{c_1}{c_2}$, the number of spare parts w^* decreases and the optimal cost$\frac{E[T(w)]}{c}$ is also decreases. Further, we observe the increasing values of p imply that optimal spare w^* becomes low and the average system cost is also go down. The effects of parameters on reliability for different values ofp are shown in Figs 1-3.

Table 1: Optimal values for r= 4, l = 5

	$\frac{c_1}{c_2}$	w^*	E [T (w^*)]	E [T (w^*)] / c_1
p = 0.5	.001	10	242.99	121.49
	.008	5	124.99	15.62
	.010	5	74.49	14.89
	.030	2	31.03	10.34
p = 0.7	.001	16	51.94	25.79
	.008	9	155.44	20.14
	.010	8	92.67	18.91
	0.030	4	44.72	14.90
p = 0.8	0.001	2	72.44	35.72
	0.008	5	771. 68	96.46
	0.010	13	134.58	26.91
	0.030	0	62.82	20.94

Table 2: Optimal values for r= 6, l = 9

	$\frac{c_1}{c_2}$	w^*	E [T (w^*)]	E [T (w^*)]/c_1
p =0 .5	0.001	7	50.41	25.07
	0.008	1	273.80	34.22
	0.010	1	148.90	29.78
	0.030	1	60.67	20.22
p = 0.7	0.001	11	52.59	26.08
	0.008	5	155.60	19.45
	0.010	4	94.46	18.89
	0.030	0	52.12	17.37
p = 0.8	0.001	22	1668.25	834.12
	0.008	12	223.84	27.48
	0.010	12	134.92	26.98
	0.030	4	61.72	20.57

Table 3: Optimal values for r= 5, l= 7

	$\frac{c_1}{c_2}$	w^*	E [T (w^*)]	E [T (w^*)] / c_1
p = 0.5	0.001	8	43.21	21.60
	0.008	3	172.90	21.61
	0.010	3	98.45	19.70
	0.030	0	44.58	14.82
p = 0.7	0.001	14	51.59	25.79
	0.008	5	161.14	20.14
	0.010	5	94.57	18.91
	0.030	2	46.46	15.48
p = 0.8	0.001	25	72.44	35.72
	0.008	14	1144.97	143.12
	0.010	11	3222.39	64.47
	0.030	6	127.93	42. 66

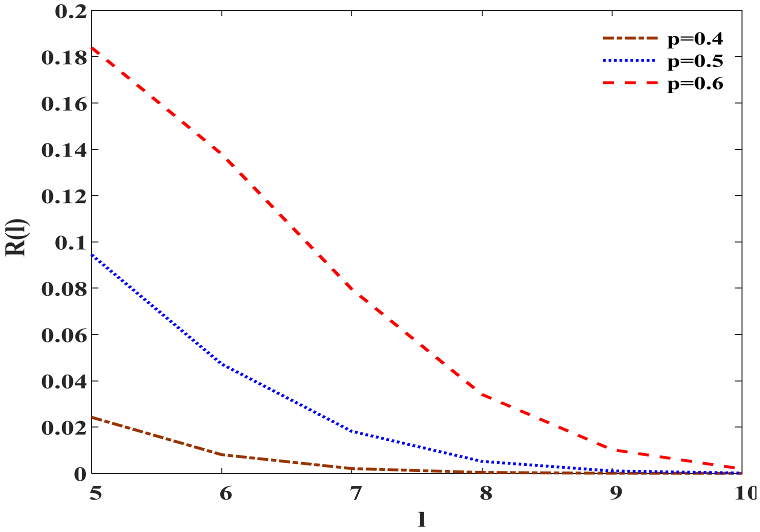

Figure 1: $Rel(l)$vs. $l$for$p = 0.4, 0.5, 0.6$

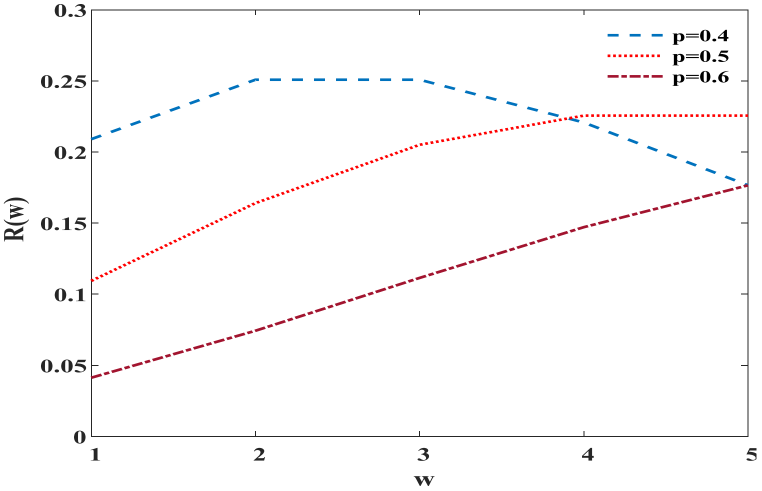

Figure 2: $Rel(w)$ vs. $w$for$p = 0.4, 0.5, 0.6$

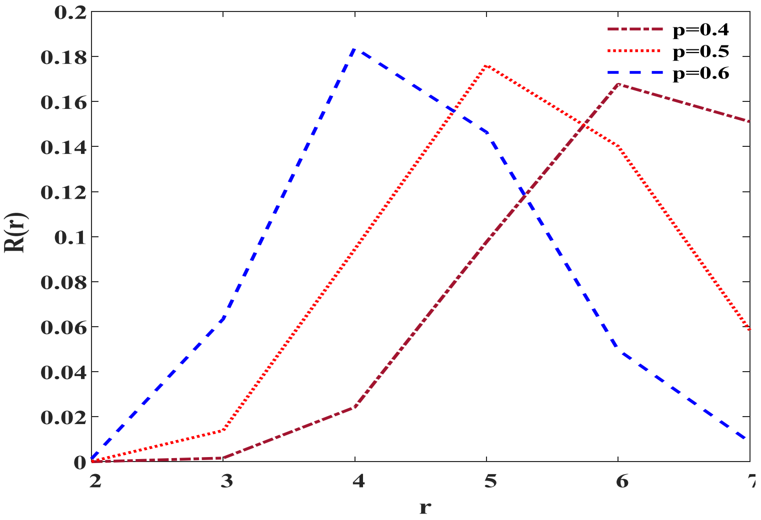

Figure 3: $Rel(r)$ vs. $r$for$p = 0.4, 0.5, 0.6$

6. CONCLUSIONS

In this paper, we have analyzed a $\left(r-l,n\right)$: G system with H.F.T.R. The determination of optimal number of spare components to minimize the mean total system cost has obtained. The cost analysis done can provide a valuable insight from the organizer's point of view. The considered model has many applications in transportation systems, communication system, distributed network, manufacturing system and can play vital role inthe design and upgradation of fault-tolerant computing systems (FTCS). Hybrid redundancy is also applicable in the reliability improvement of fault-tolerant digital systems with maximum availability and self-repairing.

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